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SEAS - WP - 2016 - 10 - 001 (For Admin only)

An Enhanced Mixed-Scaling-Rotation CORDIC Algorithm with Weighted Amplifying Factor

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Abstract (150 words, Font 12):
Mixed Scaling Rotation COordinate Rotational Digital Computer (MSR-CORDIC) algorithm has found its application in the areas where the rotation angles are known beforehand. The algorithm merges the micro-rotation and scaling operations resulting in the elimination of the overhead caused by the scaling operation. Through this paper, an improved MSR-CORDIC algorithm is proposed. This algorithm provides higher signal-to-quantization-noise ratio (SQNR) performance while preserving the features offered by the original MSR-CORDIC algorithm. The novelty of the paper lies in redefining the amplifying factor by multiplying the rotational sequences to the corresponding signed-power-of-two (SPT) terms. The proposed algorithm offers a better alternative to MSR-CORDIC without additional hardware complexity.

Keywords:
Coordinate Rotational Digital Computer (CORDIC) algorithm, Fast Fourier Transformation (FFT), Mixed Scaling Rotation (MSR)-CORDIC, Signal to Quantization Noise Ratio (SQNR), VLSI

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***Abstract*—Mixed Scaling Rotation COordinate Rotational DIgital Computer (MSR-CORDIC) algorithm has found its application in the areas where the rotation angles are known beforehand. The algorithm merges the micro-rotation and scaling operations resulting in the elimination of the overhead caused by the scaling operation. Through this paper, an improved MSR-CORDIC algorithm is proposed. This algorithm provides higher signal-to-quantization-noise ratio (SQNR) performance while preserving the features offered by the original MSR-CORDIC algorithm. The novelty of the paper lies in redefining the amplifying factor by multiplying the rotational sequences to the corresponding signed-power-of-two (SPT) terms. The proposed algorithm offers a better alternative to MSR-CORDIC without additional hardware complexity.**

***Index Terms*—Coordinate Rotational Digital Computer (CORDIC) algorithm, Fast Fourier Transformation (FFT), Mixed Scaling Rotation (MSR)-CORDIC, Signal to Quantization Noise Ratio (SQNR), VLSI.**

I. Introduction

COordinate Rotational DIgital Computer(CORDIC) is an iterative arithmetic algorithm based on the principles of two dimensional geometry. The algorithm offers simple hardware implementation consisting of shift and add operations. It is suitable for the computation of trigonometric and hyperbolic functions, multiplication and division operations and logarithms [1], [2]. The simplicity of implementing these mathematical operations leads to its applications in Digital Signal Processing, such as Fast Fourier Transformation (FFT), Eigenvalue Decomposition, Singular Value Decomposition and QR factorization [3], [4].

The iterative nature of the conventional CORDIC algorithm affects the speed of computation. Several algorithms have been proposed in the literature such as Angle Recording (AR) [5], Fast CORDIC [6], Extended Elementary Angle Set (EEAS) [7], Modified Vector Rotational (MVR) [8], Mixed Scaling Rotation [9] amongst others to reduce the number of iterations. MSR-CORDIC can also be seen as the universal vector rotational CORDIC engine encompassing aforementioned algorithms [9]. It significantly reduces the number of iterations thereby improving the speed and enhancing the signal-to-quantization-noise-ratio (SQNR) performance. It offers a unique feature of allowing intermediate vectors to have values other than unity by controlling the amplifying factor. The algorithm can be applied to the applications where the rotation angles are usually known beforehand.e.g. the twiddle factor in FFT [10].

The main theme of this paper lies in redefining the amplifying factor by introducing terms for representing the direction of the rotations. It is based on the principles of geometry consisting micro-rotations with scaling and MSR-CORDIC algorithm. With redefined amplifying factor the optimal parameters then can be calculated similar to [9] such that norm error and angle error are minimized at the same time. The main contribution of this paper lies in the fact that it provides higher SQNR performance while preserving the features of MSR-CORDIC. A strong feature of this algorithm is that it does not require additional hardware when compared to the existing MSR-CORDIC implementations.

The rest of the paper is organized as follows. A brief introduction to the MSR-CORDIC algorithm is presented next. Section II presents the main contribution of the proposed algorithm. Section III compares the simulation results of the proposed algorithm with those of MSR-CORDIC. Finally, Section IV draws the conclusions of proposed work and includes future directions.

MSR-CORDIC Scheme

MSR-CORDIC algorithm is designed such that the rotations and scaling operations are performed at the same time. Unlike the conventional CORDIC, the MSR-CORDIC algorithm minimizes the errors in both the angle and norm. It also provides the feature of adjusting the range of the norm. These unique features of MSR-CORDIC provide better SQNR performance, global solution and reduction of roundoff noise.

The algorithm 1 recalls the MSR-CORDIC scheme [9]. Various parameters are as follows: n denotes the n^{th} iteration, N denotes the total number of iterations, $\eta_i(n), \mu_j(n) \in \{-1, 0, 1\}$; $s_i(n), t_j(n) \in \{0, 1, ..., S\}$, where S denotes the number of maximum shifts; I and J denotes the number of signed-power-of-two (SPT) terms of $x(n)$ and $y(n)$ respectively; θ_n is the n^{th} elementary angle; $Z(n)$ is the accumulative angle, and $Z(0)$ is 0; $\bar{p}_n$ denotes the product of the amplifying factors in the n^{th} iteration, and $\bar{p}_0$ is 1; P denotes the scaling factor, and N_{spt} is denoted as the SPT term used which is the sum of I and J.

The design parameters $s_i(n)$, $t_j(n)$, $\eta_i(n)$ and $\mu_j(n)$ are selected such that the angle error $|Z(N) - \Theta|$ and norm error

Algorithm 1 MSR-CORDIC Scheme

1: **for** $n := 1$ to N **do**

2: Perform micro-rotations and scaling

$$\begin{bmatrix} x(n) \\ y(n) \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{I} \eta_i(n) 2^{-s_i(n)} & -\sum_{j=1}^{J} \mu_j(n) 2^{-t_j(n)} \\ \sum_{j=1}^{J} \mu_j(n) 2^{-t_j(n)} & \sum_{i=1}^{I} \eta_i(n) 2^{-s_i(n)} \end{bmatrix} \times \begin{bmatrix} x(n-1) \\ y(n-1) \end{bmatrix} \tag{1}$$

3: Calculate elementary angle

$$\theta_n = \tan^{-1}\left(\frac{\sum_{j=1}^{J} \mu_j(n) 2^{-t_j(n)}}{\sum_{i=1}^{I} \eta_i(n) 2^{-s_i(n)}} \right) \tag{2}$$

4: Update accumulation angle

$$Z(n) = Z(n-1) + \theta_n \tag{3}$$

5: Amplifying factor in the n[th] rotation

$$p_n = \sqrt{\left(\sum_{i=1}^{I} 2^{-s_i(n)} \right)^2 + \left(\sum_{j=1}^{J} 2^{-t_j(n)} \right)^2} \tag{4}$$

6: Product of the amplifying factor in the n[th] rotation

$$\bar{p}_n = \bar{p}_{n-1} \times p_n \tag{5}$$

7: **end for**

8: Scaling factor

$$P = \prod_{n=1}^{N} p_n \tag{6}$$

$|1 - P|$ are minimized at the same time; where Θ is the targeted angle.

II. The Proposed MSR-CORDIC Scheme

Given a rotation angle θ and vector $[x, y]^T$, the resultant vector $[x', y']^T$ can be computed as follows:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \tag{7}$$

The resultant angle can be decomposed of multiple angles by using the concept of micro-rotations . Hence, (7) gets modified to

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \left(\prod_{n=1}^{N} \begin{bmatrix} \cos\theta_n & -\sin\theta_n \\ \sin\theta_n & \cos\theta_n \end{bmatrix} \right) \begin{bmatrix} x \\ y \end{bmatrix}, \tag{8}$$

where,

$$\theta = \sum_{n=1}^{N} \theta_n, \tag{9}$$

N is the total number of rotations and θ_n is the n^{th} rotation angle.

The principal theme of the proposed work is to replace (8) by scaled products such that

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \left(\prod_{n=1}^{N} \begin{bmatrix} v_n \cos\theta_n & -v_n \sin\theta_n \\ v_n \sin\theta_n & v_n \cos\theta_n \end{bmatrix} \right) \begin{bmatrix} x \\ y \end{bmatrix}, \tag{10}$$

where, v_n is the scaling factor,

$$\theta = \sum_{n=1}^{N} \theta_n \tag{11}$$

and

$$\prod_{n=1}^{N} v_n = 1. \tag{12}$$

It can be observed from (10) that scaling operation is performed in parallel to the micro-rotation. It is also important to note that the intermediate vectors may have norms other than unity. Though, to retain the norm of the original vector, multiplication of all the scaling factors shall be unity. The trivial equality $cos^2\theta + sin^2\theta = 1$ and (12) can be utilized to produce

$$\prod_{n=1}^{N} \left(\sqrt{(v_n \cos\theta_n)^2 + (v_n \sin\theta_n)^2} \right) = 1. \tag{13}$$

Further, it can be observed that there is a stark similarity between the structure of MSR-CORDIC and the method of micro-rotations coupled with scaling as used in (10)–(12). The proposed novel MSR-CORDIC algorithm uses this concept in redefining the original MSR-CORDIC. Equation (1) from MSR-CORDIC is analogous to equation (10). Based on (13) the amplifying and the scaling factors defined in the algorithm 1 are modified as

$$v_n = \sqrt{\left(\sum_{i=1}^{I} \eta_i(n) 2^{-s_i(n)} \right)^2 + \left(\sum_{j=1}^{J} \mu_j(n) 2^{-t_j(n)} \right)^2} \tag{14}$$

and

$$V = \prod_{n=1}^{N} v_n. \tag{15}$$

The redefined amplifying factor contains additional terms $\mu_j(n)$ and $\eta_i(n)$. All the remaining equations (1)–(3) and (5)–(6) remain the same. The summary of the same is given in algorithm 2.

The proposed new MSR-CORDIC preserves all the features provided by the MSR-CORDIC. It can be adopted for both normal and generalized MSR-CORDIC schemes. Further, it is important to note that there is no need of any additional adders or shifters in comparison to the conventional MSR-CORDIC schemes. The boundary condition in the proposed scheme remains the same as of the classical scheme, i.e. $v_{upper} = p_{upper}$, $v_{lower} = p_{lower}$. With the same hardware

Algorithm 2 Proposed MSR-CORDIC scheme with weighted amplifying factors

for $n := 1$ to N **do**

Calculate Micro-rotations and Scaling equation, Elementary angle and Accumulation angle using equations (1), (2) and (3) respectively

Weighted amplifying factor in the n^{th} rotation

$$v_n = \sqrt{\left(\sum_{i=1}^{I} \eta_i(n) 2^{-s_i(n)}\right)^2 + \left(\sum_{j=1}^{J} \mu_j(n) 2^{-t_j(n)}\right)^2} \tag{16}$$

Product of the weighted amplifying factor in the n^{th} rotation

$$\bar{v}_n = \bar{v}_{n-1} \times v_n \tag{17}$$

where,

$$\bar{v}_0 = 1$$

end for

Scaling factor

$$V = \prod_{n=1}^{N} v_n \tag{18}$$

complexity, the proposed MSR-CORDIC provides greater SQNR performance as discussed in the next section.

III. Simulation results

A comparison of the proposed scheme and MSR-CORDIC in terms of the SQNR outcome is presented in this section. It is shown that the proposed scheme results in better SQNR performance. The constraints are fixed in the same way as outlined in [9] for the purpose of simplicity and fairness. An exhaustive search for each type of constraint is carried out to generate 512 distinct parameters sets of $\eta_i(n)$, $\mu_j(n)$, $s_i(n)$ and $t_j(n)$.

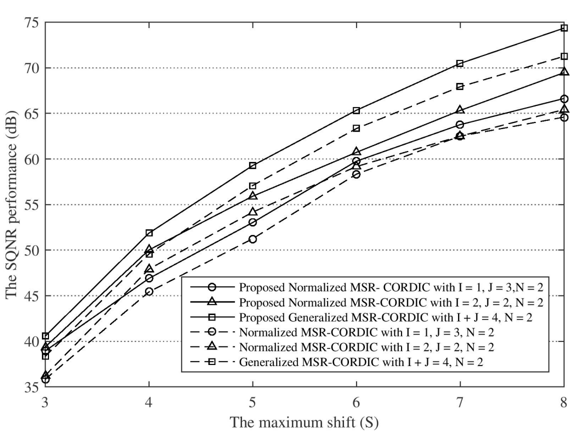

Fig. 1. SQNR Comparison between MSR-CORDIC and the proposed scheme

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MSR-CORDIC offers two sets of schemes, namely, Normalized MSR-CORDIC and Generalized MSR-CORDIC. A comparison between the proposed scheme and MSR-CORDIC is presented in Fig.1 with $N_{SPT} = 4$. Normalized scheme takes any one set of (I, J) that satisfies $I + J = 4$, i.e. $(4, 0)$, $(0, 4)$, $(1, 3)$, $(3, 1)$ and $(2, 2)$. It is important to note that sets $(4, 0)$ and $(0, 4)$ offer only scaling operation and hence they are not considered for the comparison. The SQNR performance of $(1, 3)$ and $(3, 1)$ are the same. Hence, only unique combinations such as $(1, 3)$ is taken into account for the simulations. Unlike the normalized scheme which has the fixed choice of combinations for (I, J), Generalized scheme selects the combination which minimizes the angle error $|Z(N) - \Theta|$ and norm error $|1 - V|$ the most at the same time.

The following observations can be made from Fig.1:

1) For the proposed method similar to the MSR-CORDIC, the generalized scheme offers the better SQNR performance when compared to the Normalized scheme. Furthermore, the SQNR performance of $(2, 2)$ is higher when compared to $(1, 3)$.
2) The plot shows that the proposed scheme offers higher SQNR performance for both the schemes when compared with that provided by corresponding MSR-CORDIC counterparts.

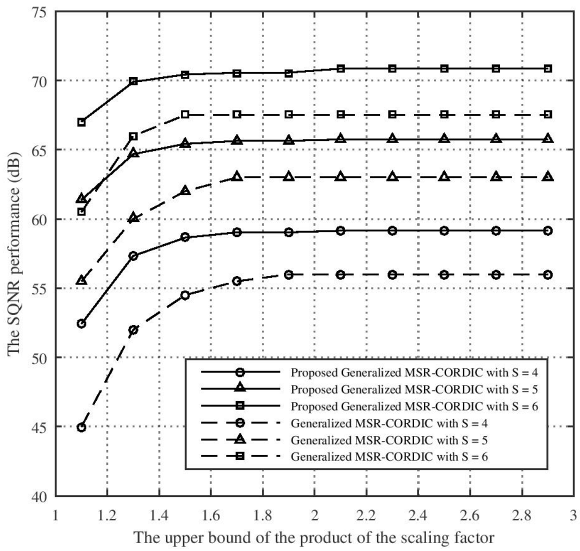
The scaling factor in the conventional CORDIC algorithm is fixed as per the number of iterations. Though, with other algorithms such as AR, MVR and EEAS the scaling factor changes with every iteration. This leads to higher roundoff noise error and hence deteriorates SQNR performance. The word length can be defined if range of the scaling factor is known *a priori*. Thus, the roundoff noise can be reduced. MSR-CORDIC allows to determine the range for the scaling factor such that $p_{lower} \leq \bar{p}_n \leq p_{upper}$ holds true. The parameter p_{lower} is fixed as $1/p_{upper}$ as per the boundary constraint explained in [9] and the same holds true for $\bar{v}_n$.

The analysis of SQNR performance with the change in scaling factor is depicted in Fig. 2. The parameters are selected as $N_{SPT} = 3$, $N = 3$ and $N_{SPT} = 4$, $N = 2$ for Fig. 2(a) and Fig. 2(b) respectively. Following can be observed from the plot:

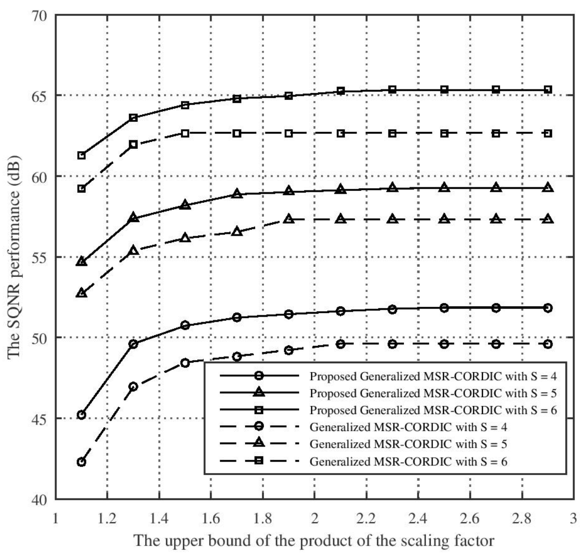
1) Similar to MSR-CORDIC, the proposed scheme saturates when v_{upper} is 1.5.
2) The proposed scheme has better SQNR performance for the same parameters. When v_{upper} value reaches 1.3, the SQNR performance of the proposed scheme is better than the saturated SQNR value of MSR-CORDIC.

The analysis of the SQNR performance with different N_{SPT} is shown in Fig. 3. For a comparison of both, conventional and proposed MSR-CORDIC schemes, the parameters are selected as $N_{SPT} = 3$ and $N_{SPT} = 4$ with $N = 2$. It can be observed that the performance of the higher N_{SPT} term is better in both the schemes. Also, the SQNR performance of the proposed scheme is better when compared with the same N_{SPT} term of the other scheme.

A further observation can be made that the hardware



(a)



(b)

Fig. 2. Comparing the relationship between SQNR performance and scaling factor value of v_{upper} in generalized scheme of MSR-CORDIC and proposed MSR-CORDIC. (a) $N_{SPT} = 3$ and $N = 3$ (b) $N_{SPT} = 4$ and $N = 2$

complexity of the proposed algorithm is the same as MSR-CORDIC since scaling and microrotation equations for both algorithms remain analogous. Hence, the proposed algorithm enhances the SQNR performance without adding hardware complexity. During the extensive and numerous simulations runs in addition to those being reported in this paper, no instance could be found where the proposed scheme resulted in inferior SQNR performance than that afforded by the conventional MSR-CORDIC algorithm.

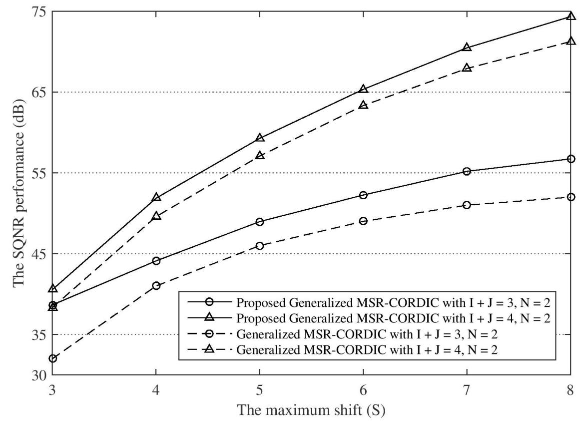


Fig. 3. Analysis of MSR-CORDIC and proposed scheme for different combinations of N_{SPT} .

IV. Conclusions

An enhanced MSR-CORDIC algorithm is proposed in this paper that employs weighted amplifying factors. The proposed algorithm can be implemented with both, Generalized and Normalized schemes. The algorithm provides better SQNR performance with no added hardware complexity. A good future direction is to perform a comparative performance analysis of the proposed scheme with widely used CORDIC methods and to study the possibilities of analytically deriving optimal values of the parameters μ, η, s and t, thereby obviating the need for extensive parameter search.

References

[1] J. E. Volder, "The CORDIC trigonometric computing technique," *IRE Transactions on Electronic Computers*, vol. EC-8, no. 3, pp. 330–334, Sept 1959.

[2] J. S. Walther, "A unified algorithm for elementary functions," pp. 379–385, 1971.

[3] P. K. Meher, J. Valls, T. B. Juang, K. Sridharan, and K. Maharatna, "50 years of CORDIC: Algorithms, architectures, and applications," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 56, no. 9, pp. 1893–1907, Sept 2009.

[4] Y. H. Hu, "CORDIC-based VLSI architectures for digital signal processing," *IEEE Signal Processing Magazine*, vol. 9, no. 3, pp. 16–35, July 1992.

[5] Y. H. Hu and S. Naganathan, "Angle recording method for efficient implementation of the CORDIC algorithm," pp. 175–178 vol.1, May 1989.

[6] J.-C. Chih and S.-G. Chen, "A fast CORDIC algorithm based on a novel angle recoding scheme," *The 2000 IEEE International Symposium on Circuits and Systems, 2000. Proceedings. ISCAS 2000 Geneva.* , vol. 4, pp. 621–624 vol.4, 2000.

[7] C.-S. Wu, A.-Y. Wu, and C.-H. Lin, "A high-performance/low-latency vector rotational CORDIC architecture based on extended elementary angle set and trellis-based searching schemes," *IEEE Transactions on Circuits and Systems II: Analog and Digital Signal Processing*, vol. 50, no. 9, pp. 589–601, Sept 2003.

[8] C.-S. Wu and A.-Y. Wu, "Modified vector rotational CORDIC (MVR-CORDIC) algorithm and architecture," *IEEE Transactions on Circuits and Systems II: Analog and Digital Signal Processing*, vol. 48, no. 6, pp. 548–561, Jun 2001.

[9] C.-H. Lin and A.-Y. Wu, "Mixed-scaling-rotation CORDIC (MSR-CORDIC) algorithm and architecture for high-performance vector rotational DSP applications," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 52, no. 11, pp. 2385–2396, Nov 2005.

[10] S. Y. Park and Y. J. Yu, “Fixed-point analysis and parameter selections of MSR-CORDIC with applications to FFT designs,” *IEEE Transactions on Signal Processing*, vol. 60, no. 12, pp. 6245–6256, Dec 2012.