Problem of the Month - October 2024 Proposed by: Dr Alok Shukla

Prove that

$$\left[\sum_{k=1}^{n} \log\left(2 - \frac{\prod_{t=1}^{t=m} \theta_{k+t}}{(\theta_k)^m + \prod_{t=1}^{t=m} \theta_{k+t}}\right)\right] \leq n - 1,$$

where $n \geq m \geq 2$, $0 < \theta_k$ for $1 \leq k \leq n$, and $\theta_{n+j} = \theta_j$ for $j = 1$ to $j = m$. Note that $\log(x)$ refers to the natural logarithm, $\log_e(x)$.