Problem of the Month - January 2026 Proposed by: Dr Alok Shukla

A research laboratory studies a device that processes *four–channel signals.* Each signal state is described by four real numbers (a_1, a_2, a_3, a_4), with the condition $a_1 a_4 - a_2 a_3 \neq 0$.

When two signal states are passed through the mixer one after the other, the resulting signal is given by the rule

$$(a_1, a_2, a_3, a_4) \diamond (b_1, b_2, b_3, b_4) \;= (a_1 b_1 + a_2 b_3,\ a_1 b_2 + a_2 b_4,\ a_3 b_1 + a_4 b_3,\ a_3 b_2 + a_4 b_4). \qquad (\star)$$

To each signal state the laboratory assigns a *stability index* $S(a_1, a_2, a_3, a_4)$, which satisfies the experimental law $S(X \diamond Y) = S(X)S(Y)$ for all signal states X, Y. The signal mixing process and the multiplicative behavior of the stability index are illustrated schematically in Figure 1. From experiments, the following facts are known:

(i) If a signal state satisfies $a_3 = 0$, then $S(a_1, a_2, 0, a_4) = |a_1|^2 |a_4|$.

(ii) The special signal $W = (0, 1, 1, 0)$ has stability $S(W) = 1$.

Using only the rules above, determine an explicit formula for $S(a_1, a_2, a_3, a_4)$ valid for all allowed signal states (a_1, a_2, a_3, a_4).

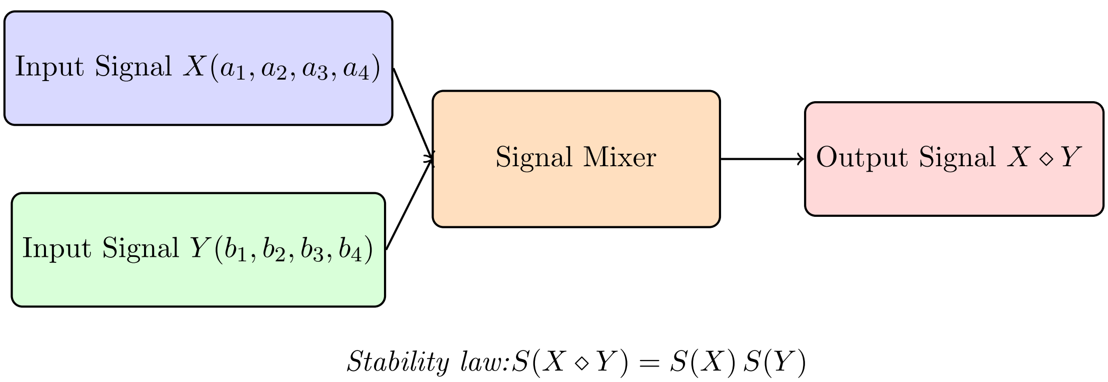

Figure 1: Schematic illustration of the signal mixing operation $X \diamond Y$ and the multiplicative behavior of the stability index.