

Let  $f$  be a differentiable function such that

$$f(x + y) = f(x) + f(y) + xy(x + y)$$

for all  $x, y \in \mathbb{R}$  and  $f(2025) = 675(2025)^2$ . Find all nonzero integer solutions of the following equation (i.e., all  $m, n, r \in \mathbb{Z} \setminus \{0\}$  that satisfy the equation), or prove that no such solutions exist:

$$\frac{f(m)}{m} + \frac{f(n)}{n} = \frac{3f(r)}{r}.$$

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