Problem of the Month - October 2023

Show that

$$\left(\prod_{n=0}^{2023}(3n+1)\right)\left(\sum_{n=0}^{2023}\,\frac{(-1)^n}{3n+1}\binom{2023}{n}\right)=3^{2023}(2023!).$$