Problem of the Month

The Greek God Dionysus decides to have an eternal festivity. On the first day of the festival only one God, the God Zeus, visits and Dionysus serves him 1 goblet of wine. On the second day Zeus visits with two other Gods and Dionysus serves $1+\frac{1}{2}$ goblets of wine dividing equally among the $1+2$ Gods. On the third day, a total of $1+2+3$ Gods, including Zeus, visit Dionysus and he serves them $1+\frac{1}{2}+\frac{1}{3}$ goblets of wine dividing it equally among them. The pattern continues forever as the Gods are immortal and there are infinitely many Gods. It means on the nth day, including Zeus, $g(n) = 1+2+3+\cdots+(n-1)+n$ Gods visit Dionysus and he serves them $w(n) = 1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}$ goblets of wine distributing it equally among them (see Table 1). How much wine does Zeus get from day 1 to day n, when in the limit $n \to \infty$?

(It is known that $\gamma = \lim_{n\to\infty} 1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}-\ln(n)$ is a finite number. You may also use the following result, which was first proved by Euler.

$$1+\frac{1}{2^2}+\frac{1}{3^2}+\cdots=\frac{\pi^2}{6}.\ )$$

Day	Number of Gods invited	Total quantity of wine served (in goblets)
1	1	1
2	$1+2$	$1+\frac{1}{2}$
3	$1+2+3$	$1+\frac{1}{2}+\frac{1}{3}$
4	$1+2+3+4$	$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}$
...	...	...
n	$1+2+3+4+\ldots+n$	$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\ldots+\frac{1}{n}$
...	...	...

Table 1: Day-wise festival details