

During the festival of Diwali, the village of Combinatorica lights up the community hall with an $n \times n$ grid of lanterns. Each lantern can glow bright (+) or dim (-) according to the following rule: For the lantern in row i and column j (both starting from 0), compute the number t as the remainder when i multiplied by j is divided by n , that is, $t = (i \cdot j) \bmod n$. If t is even, the lantern glows bright; if t is odd, the lantern glows dim.

The villagers are fascinated by the *flutteriness* of each column, the number of times the glow switches from top to bottom. Derive a formula for the flutteriness of the k -th column ($0 \leq k < n$).

Further, the Diwali display is called “dazzling” if the flutteriness of columns takes all possible values from 0 to $n - 1$. Find the condition on n for a “dazzling” Diwali display. Does $n = 2025$ result in a “dazzling” Diwali display? How about $n = 2026$?

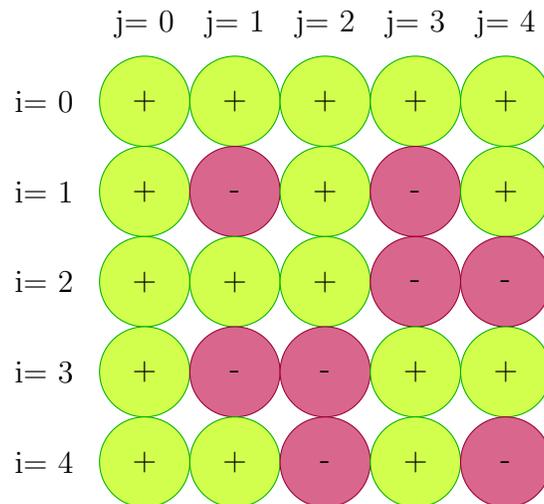


Figure 1: A cheerful Diwali lamp display in Combinatorica in a 5×5 grid pattern, where ‘+’ represents a glowing lamp and ‘-’ a dim one.
